

## Wind Study

Learning goals:

- (1) learn how to manipulate numerical data in an Excel spreadsheet,
- (2) learn how to graph two quantities to see their relationship,
- (3) learn how wind power is related to wind speed.

Background:

The kinetic energy of a single object of mass  $m$  traveling with speed  $u$  is  $\frac{1}{2}mu^2$ . Since wind is made up of many particles (air molecules) traveling with an average speed  $u$ , any given volume of moving air has an energy density (kinetic energy per volume)

$$\frac{1}{2} \frac{M}{V} u^2 = \frac{1}{2} D u^2$$

where  $D = M/V$  is the air density.

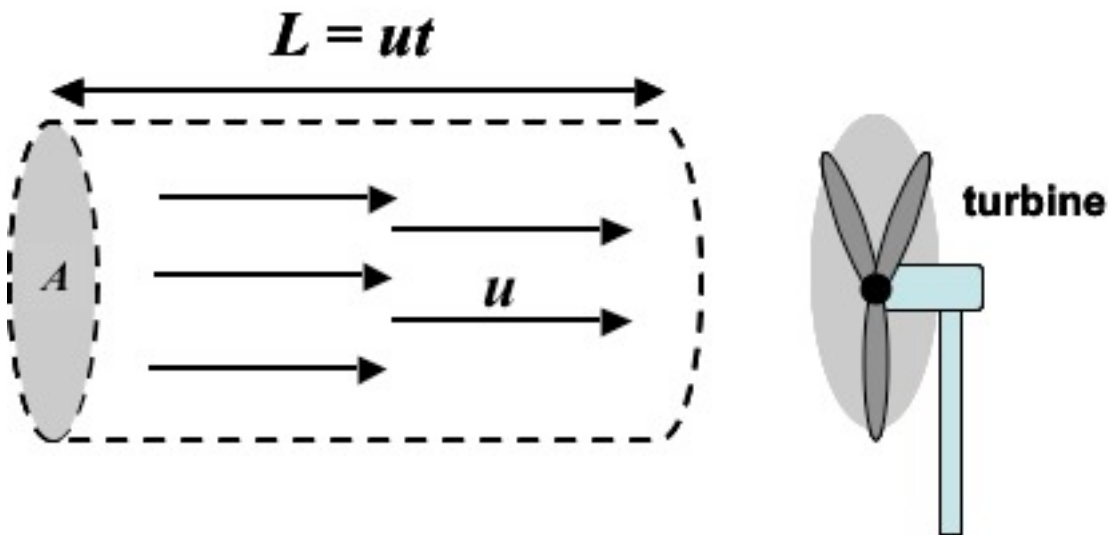


Fig. 1

Imagine then a column of air (with cross sectional area  $A$ ) traveling a distance  $L = ut$  in time  $t$ , as shown in Fig. 1. The kinetic energy inside the volume  $V = AL$  is then

$$K.E. = \frac{1}{2} D u^2 V = \frac{1}{2} D u^2 (AL) = \frac{1}{2} D u^2 (A(ut)),$$

and the available wind power (energy per time) is

$$P_{\text{wind}} = \frac{1}{2} D A u^3.$$

This is the power contained in a column air of density  $D$  and cross sectional area  $A$ , flowing at speed  $u$ . Notice that the power depends on the cube of the wind speed. This means that the power increases more dramatically with increasing wind speed. This is one of the reasons why tornadoes can be so damaging. Whenever the wind speed increases by a factor of 2, its power capacity increases by a factor of 8.

Now imagine this wind incident on the rotors of a wind turbine, as illustrated in Fig. 1. The cross sectional area of interest is then the area mapped out by the rotating blades of the turbine. If the blades are caused to rotate by the incoming wind, the outgoing wind speed just behind the turbine must be decreased. (Why? - this is conservation of energy: if the turbine rotates, it gains energy from the wind and so the wind must lose a corresponding quantity of energy).

In practice, a wind turbine does not collect all the incoming power. If it did, the wind would have to vanish behind the turbine. Instead, a turbine operates with some collection efficiency factor that determines what fraction of the incident wind power is actually converted into generated electrical power,

$$P_{\text{generated}} = f_{\text{collection}} \cdot P_{\text{wind}} = (f_{\text{collection}}) \cdot \frac{1}{2} DAu^3$$

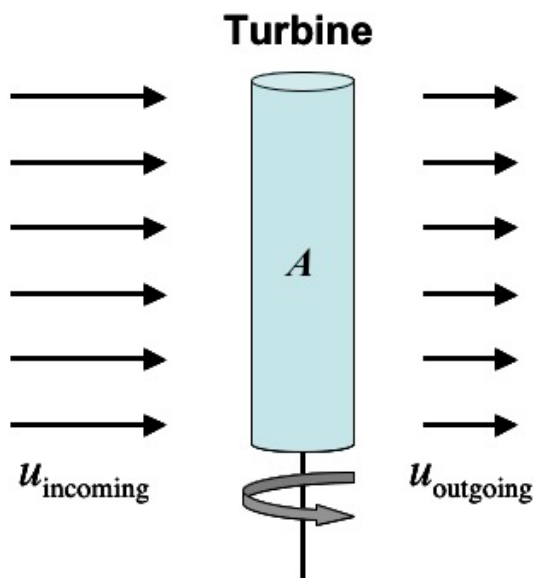


Fig. 2

Activity:

Download data from the archives for wind turbine power and wind speed and import these into an Excel spreadsheet. With Power in the first column and wind speed in the second column, you will now want to create a formula to obtain  $u^3$  to be calculated into column three. To do this, click and highlight the topmost cell in column C and type an equal sign (=) in the typesetting box that appears at the top of the page. Next, click on the topmost cell in column B (containing the first value of  $u$ ). In the typesetting box, you should now see "=B1". "B1" is the label for this cell that contains the first value of  $u$ . In the typesetting box, type "^3" next to the "B1". This instructs Excel to compute the cube of the value in cell B1 and place the result into cell C1.

To do this for the remaining values in column B, highlight cell C1 again and "copy" it. When you select copy, you will see that the border around cell C1 is now flashing. With your cursor, now select the all the cells in column C and "paste". This will repeat the formula for every row. To discontinue the flashing of cell C1, hit the "enter" key.

Now plot the power against the cube of the velocity. What should we expect? If indeed the power increases as the cube of the wind speed, we should expect to see a reasonably straight line. However, wind turbines are often designed to "shut off" if the wind is too excessive. Thus this linear relationship may break down at some value of  $u^3$ . Ask your students to determine at what wind speed the turbines are designed to shut off.

Given that each windspire turbine is vertical with a cross sectional area of  $5 \text{ m}^2$  and that the typical air density at sea level is  $1.3 \text{ kg/m}^3$ , estimate the power transfer efficiency of the turbine.